Hypothesis & Estimation

EDP 619 Week 11

Dr. Abhik Roy

Welcome!

Before moving on, please make note of the following

Welcome!

Before moving on, please make note of the following

- This serves as a toolkit of sorts and is meant for those who have taken a first semester course in descriptive & inferential statistics

Welcome!

Before moving on, please make note of the following

- This serves as a toolkit of sorts and is meant for those who have taken a first semester course in descriptive & inferential statistics
- We will address some of the basic testing approaches and estimation statistics that you likely covered before

Welcome!

Before moving on, please make note of the following

- This serves as a toolkit of sorts and is meant for those who have taken a first semester course in descriptive & inferential statistics
- We will address some of the basic testing approaches and estimation statistics that you likely covered before
- A logical step after reviewing this content is to explore the idea and need for conducting a power analysis for any statistically driven study

Welcome!

Before moving on, please make note of the following

- This serves as a toolkit of sorts and is meant for those who have taken a first semester course in descriptive & inferential statistics
- We will address some of the basic testing approaches and estimation statistics that you likely covered before
- A logical step after reviewing this content is to explore the idea and need for conducting a power analysis for any statistically driven study

As of this writing, some equations may not show up properly in Firefox. Other browsers such as Chrome and Safari do appear to render them correctly.

Essental Terms

Survey Research Methods

Statistic - Mathematical expression that describes some aspects of a set of scores for a sample

Parameter - Describes some aspect of a set of scores for a population

First a Brief Intro to Hypothesis Testing

First a Brief Intro to Hypothesis Testing

Survey Research Methods

Formally

Testing an assumption about a population parameter

Conversationally

An assumption about a particular situation of the world that is testable

Parts of a Hypothesis

Parts of a Hypothesis

The Null Hypothesis

- what is expected to happen
- must be a piece of information that is known

Parts of a Hypothesis

Survey Research Methods

The Null Hypothesis

- what is expected to happen
- must be a piece of information that is known

Notation

$$H_0$$

Parts of a Hypothesis

Survey Research Methods

The Null Hypothesis

- what is expected to happen
- must be a piece of information that is known

Notation

$$H_0$$

Alternative Hypothesis

- what else could happen
- may or may not be known

Parts of a Hypothesis

Survey Research Methods

The Null Hypothesis

- what is expected to happen
- must be a piece of information that is known

Notation

$$H_0$$

Alternative Hypothesis

- what else could happen
- may or may not be known

Notation

$$H_1 \text{ or } H_A$$

Tests of Statistical Significance

Tests of Statistical Significance

Survey Research Methods

Formally

Determination if either H_0 or H_1 can be rejected

Tests of Statistical Significance

Survey Research Methods

Formally

Determination if either H_0 or H_1 can be rejected

Conversationally

A test to figure out whether you can reasonably say if your initial assumption won't happen

Tests of Statistical Significance

Formally

Determination if either H_0 or H_1 can be rejected

Conversationally

A test to figure out whether you can reasonably say if your initial assumption won't happen

Interpretation

If results from a study goes the way that was expected, then nothing new was discovered[1]

[1] Notice that the term *unimportant* is not included within the *Interpretation*. Non results are important!

Essential Term

A **(statistical) estimation** is a sample statistic is used to estimate the value of an unknown population parameter

Positive and Negative Outcomes

Positive and Negative Outcomes

Assumption

We assume nothing out of the ordinary is going to happen - aka H_0 is expected

Positive and Negative Outcomes

Assumption

We assume nothing out of the ordinary is going to happen - aka H_0 is expected

If H_0 happens

then we have a *negative* outcome because what you expected to happen happened

Positive and Negative Outcomes

Survey Research Methods

Assumption

We assume nothing out of the ordinary is going to happen - aka H_0 is expected

If H_0 happens

then we have a *negative* outcome because what you expected to happen happened

If H_1 happens

then we have a *positive* outcome because something that was expected to happen didn't happen

Example

Example

Experiment

Over the span of one year, a group of participants with ADHD in a drug study receives a daily experimental pill that is intended to help them focus for a longer timeframe than their current medication

Example

Survey Research Methods

Experiment

Over the span of one year, a group of participants with ADHD in a drug study receives a daily experimental pill that is intended to help them focus for a longer timeframe than their current medication

If H_0 happens

The group of people *did not* report being focused for a longer timeframe than their current medication resulting in a *negative* outcome because that was an *expected* outcome

Example

Survey Research Methods

Experiment

Over the span of one year, a group of participants with ADHD in a drug study receives a daily experimental pill that is intended to help them focus for a longer timeframe than their current medication

If H_0 happens

The group of people *did not* report being focused for a longer timeframe than their current medication resulting in a *negative* outcome because that was an *expected* outcome

If H_1 happens

The group of people *did* report being focused for a longer timeframe than their current medication resulting in a *positive* outcome because that was an *unexpected* outcome

Think Big

No matter what you have heard or may be told in the future, both ***Type I Errors*** *and* ***Type II Errors*** *are not represented by a single figure, rather they each contain a different range of probabilities*

Formal Table of Statistical Error Types

Formal Table of Statistical Error Types

Decision	Null is True	Null is False
Reject Null	*False Positive* ***Type I Error***	*Correct Outcome* ***True Positive***
Fail to Reject Null	*Correct Outcome* ***True Negative***	*False Negative* ***Type II Error***

Nutshell Table of Statistical Error Types

Nutshell Table of Statistical Error Types

Survey Research Methods

You changed your mind	but it was likely the wrong decision *False Positive* ***Type I Error***	and it was likely the right decision *Correct Outcome* ***True Positive***
You didn't change your mind	and it was likely the right decision *Correct Outcome* ***True Negative***	but it was likely the wrong decision *False Negative* ***Type II Error***

Example


You're pregnant!
Type I error
(False positive)


You're not pregnant
Type II error
(False negative)

Term

Term

Alpha

- rejecting H_0 when it is true
- the probability of making a ***Type I Error***
- the chance of making a wrong decision when what was initially expected to happen actually happened

Term

Alpha	***Notation***
• rejecting H_0 when it is true • the probability of making a ***Type I Error*** • the chance of making a wrong decision when what was initially expected to happen actually happened	α

Example

If an airplane

Example

If an airplane

looks like this

then a low risk of failure - *small* α *level* - is probably acceptable

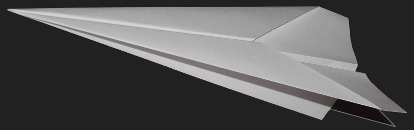
Example

If an airplane

looks like this

then a low risk of failure - *small* α *level* - is probably acceptable

looks like this



then a higher risk of failure - *large* α *level* - is probably acceptable

Term

Term

Beta

- the probability of not rejecting H_0 when it is false
- the chance associated with making a ***Type II Error***
- the possibility of making a wrong decision when something unexpected happened

Notation

β

Term

Beta

- the probability of not rejecting H_0 when it is false
- the chance associated with making a ***Type II Error***
- the possibility of making a wrong decision when something unexpected happened

Notation

$$\beta$$

Statistical Power

- the probability of not rejecting H_0 when it is false
- the chance associated with **NOT** making a ***Type II Error***
- the possibility of making the right decision when something unexpected happened

Notation

$$1 - \beta$$

Decision Making

Reality	Rejected H_0	Did Not Reject H_0
H_0 is true	*Type I Error* α *Level of Significance*	*Correct Decision* $1-\alpha$ *Level of Confidence*
H_0 is false	*Correct Outcome* $1-\beta$ *Statistical Power!*	*Type II Error* β *Rate of a Type II Error*

Decision Making

Null	$H_0 =$	Forecast says its NOT going to rain
Alternative	$H_1 =$	Something else will happen

Decision Making

Null	$H_0 =$	Forecast says its NOT going to rain
Alternative	$H_1 =$	Something else will happen

Reality	Rejected forecast	Did not reject the forecast
Forecast was right	Took an umbrella AND you're dry but may look silly or possibly fancy	Did not take an umbrella AND you're dry
Forecast was wrong	Took an umbrella AND you're dry	Did not take an umbrella AND you're wet

Note: You could have also gotten wet from snow, a flood, etc. so again ***the alternative hypothesis generally does not imply the opposite!***

Estimation

(Statistical) Estimation - a sample statistic is used to estimate the value of an unknown population parameter

Selecting a Sample Mean

Classification	Hypothesis Testing	Point/Interval Estimation
Process	Determine the probability of getting that mean if the Null is true	Estimate the value of a population mean
Outcomes	Gain information about the population mean	Gain information about the population mean

Updating Estimation for Sample Means

Updating Estimation for Sample Means

Point estimation - use of sample data to calculate a single *mean* value

Interval estimation - use of sample data to calculate a possible range of *mean* values

The Characteristic of Hypothesis Testing and Estimation

Question	Hypothesis Testing	Point/Interval Estimation
Do we know the population mean?	Yes its the Null hypothesis	No we're trying to estimate it
What is the process use dto determine?	The chance of obtaining a sample mean	The value of a population mean
What is learned?	Whether the population mean is likely correct	The range of values within which the population mean is probably contained
What is our decision?	To retain or reject the null hypothesis	No actual decison

Confidence

Confidence

Confidence Interval - an interval that contains an unknown parameter (e.g. μ) with certain degree of confidence

Level of Confidence - probability or likelihood that an interval estimate will contain an unknown population parameter

Determining the Confidence Interval

1. Calculate the *standard error of the mean*

$$\sigma_{\overline{Y}} = \frac{\sigma}{\sqrt{N}}$$

Determining the Confidence Interval

1. Calculate the *standard error of the mean*

$$\sigma_{\overline{Y}} = \frac{\sigma}{\sqrt{N}}$$

2. Decide on a *level of confidence*

Probability	**z-score**
0.90	1.645
0.95	1.96
0.99	2.576

Again its typical to have a 95% level of confidence thereby making

$$\alpha = 0.05$$

Determining the Confidence Interval (continue

3. Calculate the *confidence interval*

$$CI = \overline{Y} \pm z \cdot \sigma_{\overline{Y}}$$

Determining the Confidence Interval (continue

3. Calculate the *confidence interval*

$$CI = \overline{Y} \pm z \cdot \sigma_{\overline{Y}}$$

4. Interpret the results

Example

IQ scores in the general healthy population are approximately normally distributed with 100 ± 15. In a sample of 100 students a sample mean IQ of 103. Find the 90% confidence interval for this data.

Example

IQ scores in the general healthy population are approximately normally distributed with 100 ± 15. In a sample of 100 students a sample mean IQ of 103. Find the 90% confidence interval for this data.

Firstly we have $N = 100$, $\mu = 100$, $\sigma = 15$, and $\overline{Y} = 103$.

Example

IQ scores in the general healthy population are approximately normally distributed with 100 ± 15. In a sample of 100 students a sample mean IQ of 103. Find the 90% confidence interval for this data.

Firstly we have $N = 100$, $\mu = 100$, $\sigma = 15$, and $\overline{Y} = 103$.

1. $$\sigma_{\overline{Y}} = \frac{\sigma}{\sqrt{N}} = \frac{15}{\sqrt{100}} = 1.50$$

Example

IQ scores in the general healthy population are approximately normally distributed with 100 ± 15. In a sample of 100 students a sample mean IQ of 103. Find the 90% confidence interval for this data.

Firstly we have $N = 100$, $\mu = 100$, $\sigma = 15$, and $\overline{Y} = 103$.

1. $$\sigma_{\overline{Y}} = \frac{\sigma}{\sqrt{N}} = \frac{15}{\sqrt{100}} = 1.50$$

2. We choose a 90% level of confidence

$$z \cdot \sigma_{\overline{Y}} = 1.645 \cdot 1.50 = 2.47$$

Example

IQ scores in the general healthy population are approximately normally distributed with 100 ± 15. In a sample of 100 students a sample mean IQ of 103. Find the 90% confidence interval for this data.

Firstly we have $N = 100$, $\mu = 100$, $\sigma = 15$, and $\overline{Y} = 103$.

1. $$\sigma_{\overline{Y}} = \frac{\sigma}{\sqrt{N}} = \frac{15}{\sqrt{100}} = 1.50$$

2. We choose a 90% level of confidence

$$z \cdot \sigma_{\overline{Y}} = 1.645 \cdot 1.50 = 2.47$$

3. $$90\%\, CI = 103 \pm 2.47 = (105.47, 100.53)$$

Example

IQ scores in the general healthy population are approximately normally distributed with 100 ± 15. In a sample of 100 students a sample mean IQ of 103. Find the 90% confidence interval for this data.

Firstly we have $N = 100$, $\mu = 100$, $\sigma = 15$, and $\overline{Y} = 103$.

1. $$\sigma_{\overline{Y}} = \frac{\sigma}{\sqrt{N}} = \frac{15}{\sqrt{100}} = 1.50$$

2. We choose a 90% level of confidence

$$z \cdot \sigma_{\overline{Y}} = 1.645 \cdot 1.50 = 2.47$$

3. $$90\%\, CI = 103 \pm 2.47 = (105.47, 100.53)$$

4. So we are 90% confident that the overall mean IQ is between 100.53 and 105.47

Thats it!

If you have any questions, please reach out

Thats it!

If you have any questions, please reach out